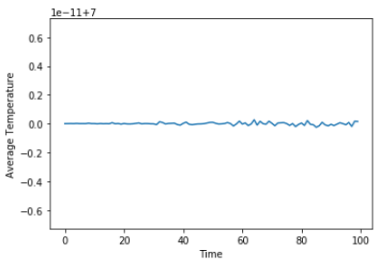
Average Temperature and Convergence Time

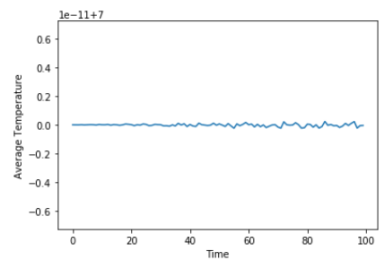
The diffusion of heat in a network is satisfied by the following equation:

$$T(t) = e^{-k\Delta t} \cdot T(0)$$

This shows that the rate of change in heat is dependent on the time and the laplacian of a matrix. To investigate this, we calculated the average temperature of a given network with respect to time. Furthermore, the networks that were compared were random and SWN to see if this had an effect on their diffusion.

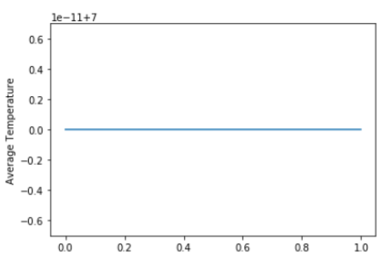


(a) Random Network

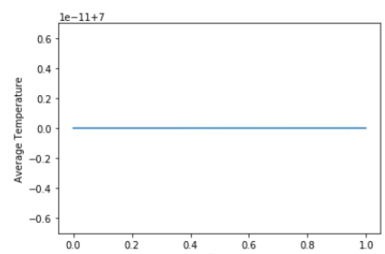


(b) Simple World Network).

Figure 1: Average Temperature vs Time for a Random Network and SWN using 1 second iterations



(a) Random Network



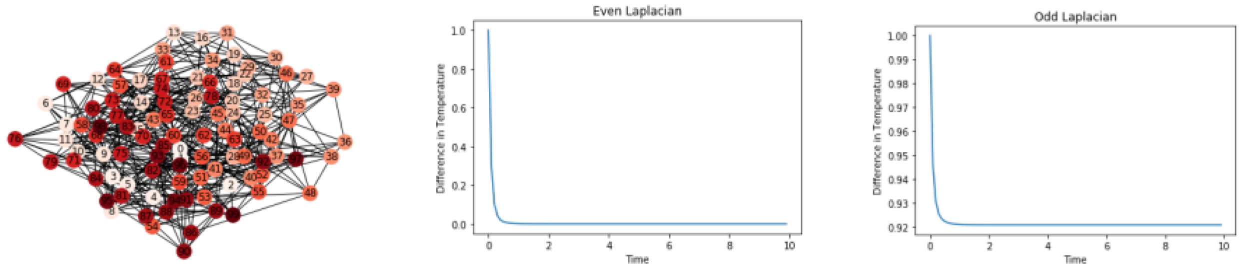
(b) Simple World Network

Figure 2: Average Temperature vs Time for a Random Network and SWN using 0.001 second iterations

Time Taken to Reach Equilibrium

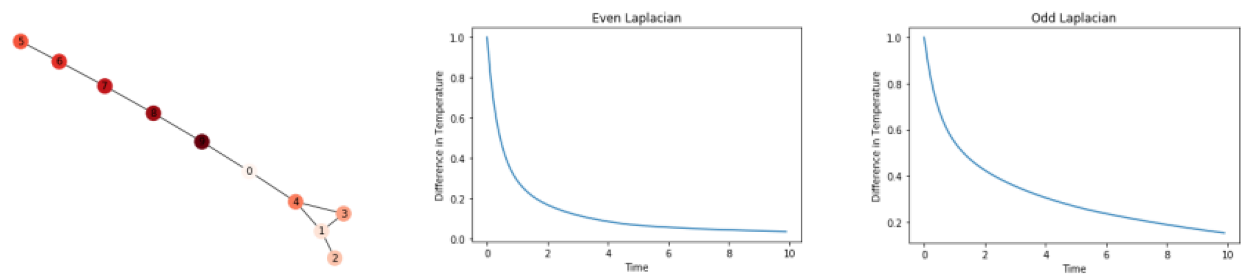
For small networks, the time taken to reach equilibrium is roughly the same if the initial temperature is concentrated on the periphery versus the center of the network.

To understand how long it takes for a network to reach equilibrium, we can find the difference between the largest and the smallest temperature in any given network. That way, even if the network has a disconnect, the temperature difference will still converge to some value. This is shown in figure for a network of 100 nodes, using both the even and odd laplacian.



(a) SWN 100 nodes (b) Temperature Convergence with Even Laplacian (c) Temperature Convergence with Odd Laplacian

Figure 3: Temperature Convergence for a Small World Network



(a) SWN 10 nodes (b) Temperature Convergence with Even Laplacian (c) Temperature Convergence with Odd Laplacian

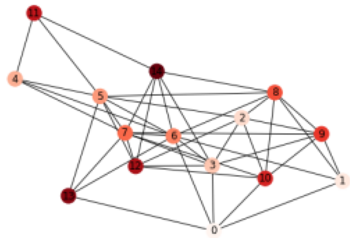
Figure 4: Temperature Convergence for a Small World Network

In figure 3, the temperature for convergence for both even and odd laplacian is fairly similar, however, there are situations where the two times are totally different such as in figure 4. By comparing these two networks, we can see that when the network is large, the temperature will decrease more rapidly due to the larger number of nodes.

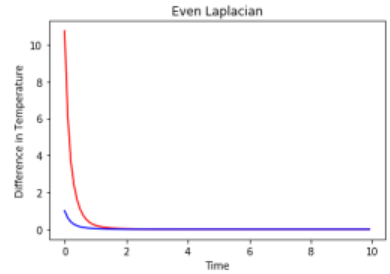
Claim - For larger networks, there is negligible difference between temperature diffusion using even and odd laplacians, however, the difference becomes notable when using smaller networks. However, this also depends on the small worldness of the graphs.

Comparing the Convergence Time within a Network

From the previous sections, when analysing the time taken for a network to reach equilibrium, it seemed as though it was roughly the same when the temperature was initially concentrated on the periphery versus on the center. This showed that the diffusion of a network was actually dependent of the energy of the initial state. To investigate this, I compared the diffusion of a network with an initial state of 1 followed by zeros, versus a network with an initial state of its eigenvalues. This is shown in figure 5.



(a) SWN



(b) Temperature Difference vs Time

Figure 5: Average Temperature vs Time for a SWN using eigenvalues (red) and a different initial state (blue)

<u>Fourier Basis</u>

To investigate a network where the initial state is one of the eigenvalues, I changed the basis of the laplacian. This is called its Fourier basis. To do this, the inverse of the eigenvectors is dot multiplied with the vector of eigenvalues. This gives a matrix, with the diagonal equal to the eigenvalues of the network.