Complex Networks: Scale-Free and Spatial Networks

Scale-Free Networks

Recall: The degree distribution of a network is the set of values that describes the degree of each node in a network.

Definition 1. A scale-free network is described as any network whose degree distribution follows a power law. This means that the proportion of nodes in the network, $P(k)$ that are connected to k number of nodes, is given as follows:

$$P(k) \propto k^{-\gamma}, \gamma > 1$$

Examples of scale-free networks: The world-wide web, Electronic Circuits

Power Law Distribution

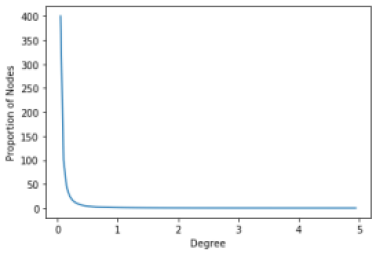

Figure 1: Power Law Distribution $\gamma = 2$

For scale-free networks, nodes that are already connected to a large number of nodes, are more likely to gain even more connections compared with nodes that have a lower degree. This is shown from the degree distribution following a power law distribution curve in figure 1. For this reason, scale-free networks usually have large hubs, and as the network increases in size, the underlying structure of the network remains the same. This is shown in figure 2. Thus, the characteristics of this kind of network is independent of the size of the network, meaning it is "scale free".

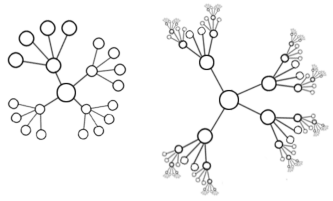

Figure 2: Smaller and Larger Scale-Free Graph
(Groningen 2016)

Difference between Scale-free, Random and SWN

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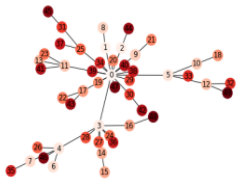
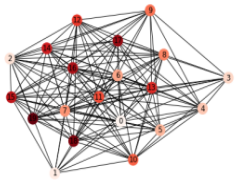
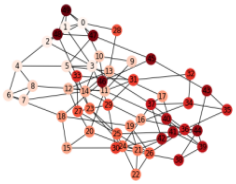
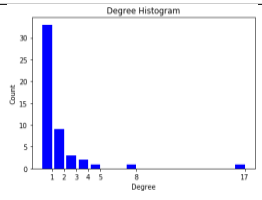
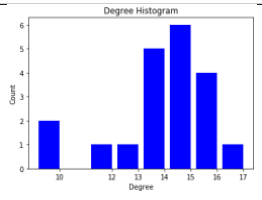
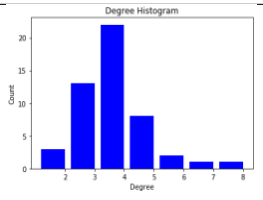
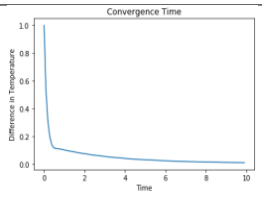
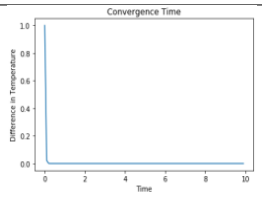
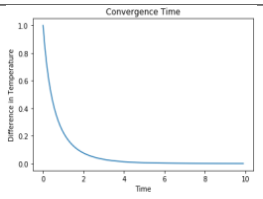
	Scale-Free Networks	**Random Networks**	**Small World Networks**
Example	Barabasi-Albert	Erdos-Renyi	Watts-Strogatz
Visualization			
Degree Distribution			
Convergence Time			
Equations	$P(k) \propto k^{-\gamma}$	Random	$L \propto logN$

*L is the average distance between nodes and N is the number of nodes in the network.

Spatial Networks

Spacial networks are networks in which the nodes and edges are arranged in space according to a certain metric. Examples include lattice graphs and random graphs. In this case, a graph with N nodes will have two nodes connected if the distance between them is smaller than a given value. Power grids and road maps are examples of real life spatial networks.

Characteristics of Spacial Networks

1. Planar Networks: These are graphs that can lay flat on a plane. All planar graphs can be represented by spatial networks, however, the reverse is not true.
2. Voronoi Tessellation: Spatial networks can be represented as Voronoi diagrams. These are diagrams that represent a network by dividing it into regions that hold nodes.
3. Space and Topology: These can be measured using the adjacency matrix, laplacian, omega and sigma indices, etc.

Any network in which the nodes are arranged according to a certain metric, can be said to be a spatial network.

Bibliography

Groningen, University of (2016). *Scale-Free Networks*. URL: https://www.futurelearn.com/courses/complexity-and-uncertainty/0/steps/1855.